Exercises – Serie 1 – Review of fluid dynamics

Exercise 1: Flow around an airfoil

We consider a flow around an airfoil with $1\ m$ chord length, which moves at $U_\infty = 30\ ms^{-1}$ in air at ambient pressure p_∞. The kinematic viscosity of air is $\nu_{air} = 15 \times 10^{-6}\ m^2/s$.

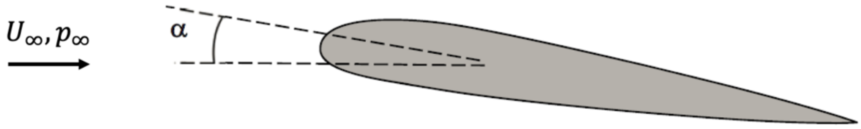

Figure 1: Flow around an airfoil.

1. Is it possible to consider the flow as incompressible?
2. Give the expression of the pressure coefficient (C_p) and draw its evolution along the pressure and suction sides of the airfoil.
3. How do these C_p curves change if the velocity is increased from 30 to 60 ms^{-1} with the incidence angle unchanged?
4. What is the value of C_p at the stagnation point? Justify.
5. Draw the lift and drag forces (vectors) on the airfoil.
6. Give the expression of the lift and drag coefficients (C_L and C_D).
7. Draw the evolution of C_L and C_D as a function of incidence angle.
8. What become C_L and C_D for a similar airfoil with $0.1\ m$ chord length, which moves in water at $20\ ms^{-1}$ velocity for a given incidence angle? (Kinematic viscosity: $\nu_{water} = 10^{-6}\ m^2/s$).

Let's consider an airplane in steady and level (horizontal) flight and given maximum lift coefficient $C_{L,max}$, minimum drag coefficient $C_{D,min}$, maximum thrust T_{max}, weight W, a lift L, area S, and air density ρ_∞.

9. Derive the stall velocity (i.e. the velocity at which level flight cannot be maintained).
10. Derive the maximum velocity of the airplane.

Exercise 2: Wind Tunnel Testing

1. An Airbus $A380$ has an $80\ m$ wingspan and is designed to cruise at a Mach number of $M = 0.85$ where the air temperature and pressure are $-40°C$ and $0.25\ bar$. The flow field around the aircraft is tested in a wind tunnel using an $80\ cm$ wingspan model. The air can be considered as an ideal gas ($p = \rho R_{air} T, R_{air} = 287\ \mathrm{JK^{-1}kg^{-1}}$) and the relation for sound speed is $a = \sqrt{\gamma R_{air} T}$, $\gamma = 1.4$. We assume that viscosity is entirely independent from pressure and density and can be found on tables online.

a) If the temperature inside the wind tunnel is maintained at $20°C$, propose a combination of wind tunnel velocity and pressure which would enable adequate modeling of the effects of viscosity and compressibility.

b) If we used a water tunnel, why both conditions cannot be met at same time? Which flow conditions would be required for an adequate modeling of the effects of viscosity?

2. To accurately estimate the drag and overturning moment experienced by a rocket on its launch pad due to wind, wind tunnel tests are carried out. The goal is to determine the loading on the rocket at a wind speed of $50\ ms^{-1}$. A $1/15^{th}$scale model of the rocket is placed in a wind tunnel operating at 20 atmospheres and at atmospheric temperature.

a) Using dynamic similarity between the wind tunnel flow and the full-scale rocket flow, calculate the required airspeed in the wind tunnel. State any assumptions.

b) Given that the drag and overturning moment measured on the model at the calculated airspeed are $5.2\ kN$ and $4.6\ kN.m$, respectively, calculate the corresponding drag force and overturning moment acting on the full-scale rocket at a wind speed of $50\ ms^{-1}$.

Exercise 3: Dimensional analysis

1. Determine the non-dimensional groups that the lift of a wing, L, depends on.
   (Hint: list the variables that the lift depends on, construct the dimensionless form of the lift and use the Buckingham-π theorem to find the dimensionless groups).
2. Dimensional analysis of the boundary layer.

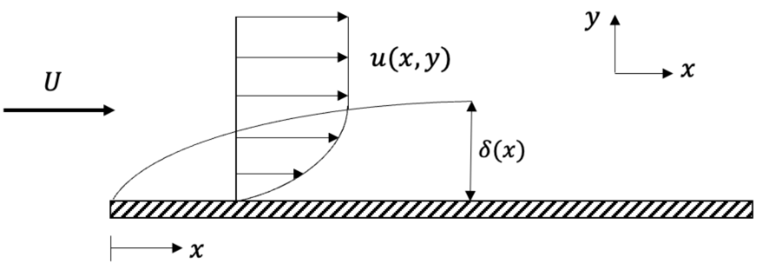

Figure 2: Laminar boundary layer on a flat plate.

2D laminar boundary layer equation (x-momentum and continuity)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

a) Using the laminar boundary layer equation and scaling arguments, explain why the thickness, δ, of a laminar boundary layer normalized with the streamwise length scale L is proportional to the inverse of the square root of the Reynolds number, i.e.,

$$\frac{\delta}{L} \propto \sqrt{\frac{\nu}{UL}} = \frac{1}{\sqrt{Re_L}}$$

where U is the freestream velocity (constant) and ν is the kinematic viscosity.

b) The analytical solution derived by H. Blasius (1908) for the laminar boundary-layer thickness of a horizontal flat plate, δ, defined as the value of y for which $u/U = 0.99$ is:

$$\delta = \frac{4.9x}{\sqrt{Re_x}}$$

Where x is the streamwise distance from the leading edge of the plate.
Using the programming language of your choice, compute and plot the evolution of the boundary layer height of an airflow over a flat plate for different freestream velocities U .